The Mathenauts

BY NORMAN KAGAN

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They sailed the star lanes in a mathematical abstraction—and reached an unbelievable goal!

I

It happened on my fifth trip into the spaces, and the first ever made under the private enterprise acts. It took a long time to get the P.E.A. through Congress for mathenautics, but the precedents went all the way back to the Telstar satellite a hundred years ago, and most of the concepts are in books anyone can buy, though not so readily understand. Besides, it didn't matter if BC-flight was made public or not. All mathenauts are crazy. Everybody knows that.

Take our crew. Johnny Pearl took a pin along whenever he went baby-sitting for the grad students at Berkeley, and three months later the mothers invariably found out they were pregnant again. And Pearl was our physcist.

Then there was Goldwasser. Ed Goldwasser always sits in those pan-on-a-post cigarette holders when we're in New York, and if you ask him, he grumbles; "Well, its an ash-

tray, ain't it?" A punster and a pataphysicist. I would never have chosen him to go, except that he and I got the idea together, so I couldn't say no.

Ted Anderson was our metamathematician. He's about half a nanosecond behind Ephriam Cohen (the co-inventor of BC-flight) and has about six nervous breakdowns a month trying to pass him. But he's got the best practical knowledge of the BC-drive outside Princeton, if practical knowledge means anything with respect to a pure mathematical abstraction.

And me — topologist. A topologist is a man who can't tell a doughnut from a cup of coffee. (I'll explain that some other time.) Seriously, I specialize in some of the more abstruse properties of geometric structures. "Did Galois discover that theorem before or after he died?" is a sample of my conversation.

Sure, mathenauts are mathenuts. But as we found out, not quite mathenutty enough.

The ship, the *Abrecht Dold,* was a twelve googol scout that Ed Goldwasser and I'd picked up cheap from the N.Y.U. Courant Mathematical Sciences Institute. She wasn't the Princeton I.A.S. *Von-Neumann,* with googolplex coils and a chapter of the D.A.R., and she wasn't one of those new toys you've been seeing for a rich man and his grandmother. Her coils were DNA molecules, and the psychosomatics were straight from the Brill Institute at Harvard. A sweet ship. For psychic ecology we'd gotten a bunch of kids from the Bronx College of the New York City University, commonsense types — business majors, engineers, pre-meds. But kids.

I was looking over Ephraim Cohen's latest paper, *Nymphomaniac Nested Complexes With Rossian Irrevelancies* (old Ice Cream Cohen loves sexy titles), when the trouble started. We'd abstracted, and Goldwasser and Pearl had signalled me from the lab that they were ready for the first tests. I made the *Dold* invariant, and shoved off through one of the passages that linked the isomorphomechanism and the lab. (We kept the ship in free fall for convenience). I was about half way along the tube when the immy failed and the walls began to close in.

Someone (probably me) had scavenged the grablines to cut up for extra connectivity. I spread my legs and braked against the walls of the tube, believing with all my might. On second thought I let the walls sink in and braked with my palms. It would've been no trick to hold the walls for awhile. Without the immy my own imagination would hold them, this far from the B.C.N.Y. kids. But that might've brought more trouble — I'd probably made some silly mistake, and the kids, who might not notice a simple contraction or shear, would crack up under some weirdomorphism. And if we lost the kids...

So anyway I just dug my feet in against the mirage and tried to slow up, on a surface that no one'd bothered to think any friction into. Of course, if you've read some of the popular accounts of math-sailing,

you'd think I'd just duck back through a hole in the fiftieth dimension to the immy. But it doesn't work out that way. A ship in BC-flight is a very precarious structure in a philosophical sense. That's why we carry a psychic ecology, and that's why Brill conditioning takes six years, plus with a Ph.D. in pure math, to absorb. As it was, there weren't a hundred mathenauts yet, and we'd been lucky to get Pearl and Anderson, despite their personalities. Anyway, a mathenaut should never forget his postulates, or he'll find himself floating in 27-space, with nary a notion to be named.

Then the walls really did vanish — NO! — and I found myself at the junction of two passages. The other had a grabline. I caught it and rebounded, then swarmed back along the tube to the control room.

About then the walls really did begin to cave in, till after ten seconds I was climbing down into a funnel. I caught my breath, swallowed some Dramamine, and burst into the control room.

The heart of the ship was pulsing and throbbing. For a moment I thought I was back in Hawaii with my aqualung, an invader in a shifting, shimmering world of sea fronds and barracuda. But it was no immy, no immy — a rubber room without the notion of distance that we take for granted (technically, a room with topological properties but no metric ones). To my eyes, it was as if everything had been transmuted to the finest, most pliable rubber, and chose to shift or swell as my gaze met it. Instrument racks and chairs and books shrank and ballooned and twisted, and floor and ceiling vibrated with my breath. It was as if I had to keep thinking about everything for it to remain still.

It was horrible.

Ted Anderson was hanging in front of the immy, the isomorphomechanism, but he was in no shape to do anything. In fact, he was in no shape at all. His body was pulsing and throbbing and shaking, so his hands were too big or too small to manipulate the controls, or his eyes shrank or blossomed. Poor Ted's nerves had gone again.

I shoved against the wall and bulleted toward him, a fish in a weaving, shifting undersea landscape, concentrating desperately on my body and the old structure of the room. (This is why physical training is so important). For an instant I was choking and screaming in a hairy blackness, a nightmare inside-out total inversion; then I was back in the control room, and had shoved Ted away from the instruments, cursing when nothing happened, then bracing against the wall panels and shoving again. He drifted away.

The immy was all right. The twiddles circuits between the B.C.N.Y kids and the rest of the Dold had been cut out. I set up an orthonormal system and punched the immy.

Across the shuddering, shifting room Ted tried to speak, but found it too difficult. Great Gauss, he was lucky his aorta hadn't contracted to a straw and given him a coronary! I clamped down on my own circu-

latory system viciously, while he struggled to speak. Finally he kicked off and came tumbling toward me, mouthing and flailing his notebook.

I hit the circuit. The room shifted about and for an instant Ted Anderson hung, ghostly, amid the isomorphomechanism's one - to - ones. He disappeared!

II

The invention of BC-flight was the culmination of a century of work in algebraic topology and experimental psychology. For thousands of years men had speculated as to the nature of the world. For the past five hundred, physics and the physical sciences had held sway. Then Thomas Brill and Ephraim Cohen peeled away another layer of the reality onion, and the spaces-sciences came into being.

If you insist on an analogy — well, a scientist touches and probes the real universe, and abstracts an idealization into his head. Mathenautics allows him to grab himself by the scruff of the neck and pull himself up into the idealization. See — I *told* you.

Okay, we'll try it slowly. Science assumes the universe to be ordered, and investigates the nature of the ordering. In the "hard" sciences, mathematics is the basis of the ordering the scientist puts on nature. By the twentieth century, a large portion of the physical processes and materials in the universe were found to submit to such an ordering (e.g.: analytic mechanics and the motions of the planets). Some scientists were even applying mathematical structures to aggregates of living things, and to living processes.

Cohen and Brill asked (in ways far apart), "If order and organization seem to be a natural part of the universe, why can't we remove these qualities from coarse matter and space, and study them separately?" The answer was BC-flight.

Through certain purely mathematical "mechanisms" and special psychological training, selected scientists (the term "mathenaut" came later, slang from the faddy "astronautics" that was current at the time), could be shifted into the abstract.

The first mathenautical ships were crewed with young scientists and mathematicians who'd received Tom Brill's treatments and Ephraim Cohen's skullcracking sessions on the BC-field. The ships went into BC-flight and vanished.

By the theory, the ships didn't *go* anywhere. But the effect was somehow real. Just as a materialist might *see* organic machines instead of people, so, multiplied a millionfold, the mathenauts saw the raw mathematical structure of space — Riemann space, Hausdorf space, vector space — without matter. A crowd of people existed as an immensely complicated *something* in vector space. The study of these *somethings* was yielding immense amounts of knowledge. Pataphysics, patasociology, patapsychology were wild, baffling new fields of knowledge.

But the math universes were strange, alien. How could you learn

to live in Flatland? The wildcat minds of the first crews were too creative. They become disoriented. Hence the immies (short for isomorphomechanisms) and their power supplies — SayCows, DaughtAmSayCows, DaughtAmRevs, the B.C.N.Y. kids — fatheads, stuffed shirts, personality types that clung to commonsense where there was none, and preserved (locally) a ship's psychic ecology. Inside the BC-field, normalcy. Outside, raw imagination.

Johnny, Ted, Goldy and I had chosen vector spaces with certain toplogical properties to test Goldy's commercial concept. Outside the BC-field there was dimension but no distance, structure but no shape. Inside —

"By Riemann's tensors!" Pearl cried.

He was at the iris of one of the tubes. A moment later Ed Goldwasser joined him. "What happened to Ted?"

"I — I don't know. No — yes I do!"

I released the controls I had on my body, and stopped thinking about the room. The immy was working again. "He was doing something with the controls when the twiddles circuits failed. When I got them working again and the room snapped back into shape, he happened to be where the immy had been. The commonsense circuits rejected him."

"So where did he go?" asked Pearl.

"I don't know."

I was sweating. I was thinking of all the things that could've happened when we lost the isomorphomechanism. Some subconscious twitch and you're rotated half a dozen dimensions out of phase, so you're floating in the raw stuff of thought, with maybe a hair-thin line around you to tell you where the ship has been, or maybe two polar dots. Or the ship takes the notion to shrink pea-size, so you're squeezed through all the tubes and compartments and smashed to jelly when we orthonormalize. Galois! We'd been lucky.

The last thought gave me a notion "Could we have shrunk so we're inside his body? Or he grown so we're floating in his liver?"

"No," said Goldy. "Topology is preserved. But I don't — or, hell — I really don't know. If he grew so big he was outside the psychic ecology, he might just have faded away . . ."

The big pataphysicist wrinkled up his face inside his beard. "*Alice* should be required reading for mathenauts," he muttered reflectively. "The real trouble is no one has ever been outside the psychic ecology when a ship is in BC-flight, and been back to tell about it. The animal experiments and the *Norbert Wiener* just never returned. And there were people, like Wilbur on the *Paul R. Halmos*. He just disappeared."

"But you work outside the field," said Pearl, in a high, desperate voice. The psychist looked pale.

"No," said Goldy. "I use, well — 'instruments', powered from the immy."


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"Who knows what could've happened?" I said, growing nervous myself. "You know you can map the volume of a sphere into the whole universe using the ratio: IR: R equals R: OR where IR and OR are the inside and outside distances for the points. Maybe that's what happened to Ted. Maybe he's just outside the ship, filling all space with his metamath and his acne?"

"Down boy," said Goldwasser. "I've got a simpler suggestion. Let's check over the ship, compartment by compartment. Maybe he's in it somewhere, unconscious."

But he wasn't on the ship.

We went over it twice, every tube, every compartment. (In reality, a mathenautic ship looks like a radio, ripped out of its case and flying through the air. We ended up in the ecology section, a big Broadway line subway car that roared and rattled in the middle of darkness in the middle of nothing. The B.C.N.Y. kids were all there—Freddi Urbont clucking happily away to her boy friend, chubby and smily and an education major; Byron and Burbitt, electronics engineers, ecstatic over the latest copy of *C-Quantum*; Stephen Seidmann, a number theory major, quietly proving that since Harvard is the best school in the world, and B.C.N.Y. is better than Harvard, that B.C.N.Y. is the best school in the world; two citizens with nose jobs and names I'd forgotten, engaged in a filthy discussion of glands and organs and meat. The walls were firm, the straw seats scratchy and uncomfortable. The projectors showed we were just entering the 72nd Street stop. How real, how comforting! I slid the door open to rejoin Johnny and Ed. The subway riders saw me slip into freefall, and glimpsed the emptiness of vector space.

Hell broke loose!

The far side of the car bulged inward, the glass smashing and the metal groaning. The poor CUNYs had no compensation training!

Freddi Urbont burst into tears. Byron and Burbitt sqawcked and yelled as the bubble in the floor swallowed them. The wall next to the nose jobs sprouted a dozen phallic symbols, while the seat bubbled with breasts. The walls began to melt. Seidmann began to yell about the special status of N. Y. City University Honors Program students. All was in confusion, and my chest froze as I realized the psychic ecology was about to collapse.

Pearl acted with a speed and a surety I'd never have imagined. He shoved me out of the way and launched himself furiously at the other end of the car, now in free fall. There he pivoted, smiled horribly and at the top of his lungs began singing *The Purple and the Black*.

Goldy and I had enough presence of mind to join him. Concentrating desperately on the shape and form of the car, we blasted the air with our devotion to Sheppard Hall, our love of Convent Avenue and our eternal devotion to Lewissohn Sta-

dium. Somehow it saved us. The room rumbled and twisted and reformed, and soon the eight of us were back in the tired old subway car that brought its daily catch of Beavers to 139th Street.

The equilibrium was still precarious. I heard Goldwasser telling the nosejobs his terrible monologue about the "Volvo I want to buy. I can be the first to break the door membranes, and when I get my hands on that big, fat steering wheel, ohh!, it'll be a week before I climb out of it!"

Pearl was cooing to Urbont how wonderful she was as the valedictorian at her Junior High, how great the teaching profession was, and how useful, and how interesting.

As for me; "Well, I guess you're right, Steve. I should have gone to B.C.N.Y. instead of Berkeley."

"That's right, Jimmy. After all, B.C.N.Y. has some of the best number theory people in the world. It's great to be a number theory major at B.C.N.Y. It's great to mumble and shout about infinite congruences and relative primes and know no one understands you, and wouldn't be interested if they did."

"I guess you're right, Steve."

"That's right, Jimmy. B.C.N.Y. has some of the greatest educators in the world. Like Dean Cashew, who started the Privileged Student Program. It sure is wonderful. At Berkeley you're just another student, but at B.C.N.Y. you can look down and laugh at all the people not in the Privileged Student Program."

"I guess you're right, Steve."

"That's right, Jimmy. At B.C.N.Y. we have people that've turned down full scholarships to Chicago and M.I.T. Of course, I had to chop off — I mean they're quadruple amputees, but they love every minute down at the clinic."

— and so on.

When the scrap paper and the gum wrappers were up to our knees and there were four false pan handlers in the car, Johnny called a halt. The little psychist smiled and nodded as he walked the three of us carefully out the door.

"Standard technique," he murmured to no one in particular. "Doing *something* immediately rather than the best thing a while later. Their morale was shot, so I —" He trailed off.

"Are they really that sensitive?" Goldwasser asked. "I thought their training was better than that."

"You act like they were components in an electronics rig," said Pearl jerkily. "You know that remedial sensory perception, the ability to percieve the dull routine that normal people ignore, is a very delicate talent. Don't you realize we mustn't interrupt their concentra with orthodox things, and especially now with the extraordinary!"

Pearl was well launched. "In the dark ages such people were called dullards and subnormals. Only now, in our enlightened age, do we realize their true ability to know things outside the ordinary senses — a talent vital for BC-flight."

The tedium and meaninglessness of life which we rationalize away —

"A ship is more mind than matter, and if you upset that mind —

Amm. I could hypnotize them. I could, but it fogs the sense of reality. The sense of reality is very important to —"

He paled suddenly, and the corridor walls began to stutter. "I, I think I'd better stay with them," he said quickly. "I know how to coach them, and it's better —"

He flung open the door and went back into the coach. Goldwasser and I looked at each other. Pearl was a trained mathenaut, and he'd acted bravely when the CUNYs had panicked. But his specialty was people, not paramath. Well, let him stay where he was most effective.

"Let's check the lab," I muttered.

Neither of us spoke as we moved toward the lab — slap a wall, pull yourself forward, twist round some instrumentation — the "reaction swim" of a man in free fall. The walls began to quiver again, and I could see Goldy clamp down on his body and memories of this part of the ship. We were nearing the limits of the BC-field. The lab itself, and the experimental apparatus, stuck out into vector space.

"Let's make our tests and go home," I told Goldy. "This trip has too much of the stink of failure about it."

Neither of us mentioned Ted as we entered the lab.

III

Remember this was a commercial project. We weren't pata-sociologists studying abstract groups, or super-purists looking for the first point. We wanted money.

Goldy thought he had a quick moneymaking scheme for us, but Goldy hasn't been normal since he took Polykarp Kusch's "Kusch of Death" at Columbia, "Electrodimensions and Magnespace." He was going to build four-dimensional molecules.

Go back to Flatland. Imagine a hollow paper pyramid on the surface of that two-dimensional world. To a Flatlander, it is a triangle. Flop down the sides — four triangles. Now put a molecule in each face — one molecule, four molecules. Neat! And recall that you have infinite dimensions available. Think of the storage possibilities alone. All the books of the world in a viewer, all the food in the world in your pack. A television the size of a piece of paper; circuits looped through dim-19. Loop an entire industrial plant through hyperspace, and get one the size and shape of a billboard. Shove raw materials in one side — pull finished products out the other!

But how do you make 4-dim molecules? Goldy thought he had a way, and Ted Anderson had checked over the math and pronounced it workable. The notion rested in the middle of the lab: a queer, half-understood machine of mind and matter called a Grahm-Schmidt generator.

"Jeez, Ed! This lab looks like your old room back in Diego Borough."

"Yeah," said Goldwasser. "Johnny said it would be a good idea. Orientation against *that*."

That was the outside of the lab,

raw topological space, without energy or matter or time. It was the shape and color of what you see in the back of your head.

I looked away.

Goldwasser's room was a duplicate of his old home — the metal desk, the electronics rigs, the immense bookshelves, half-filled with with physics and half with religious works. I picked up a copy of Stacé's *Time and Eternity* and thumbed through it, then put it down, embarrassed.

"Good reading for a place like this." Goldwasser smiled.

He sat down at the desk and began to check out his "instruments" from the locked drawer where he'd kept them. Once he reached across the desk and turned on a tape of Gene Gerard's *Excelsior!* The flat midwestern voice murmured in the background.

"First, I need some hands," said Ed. "So —"

Out in the nothingness two pairs of lines met at right angles. For an instant, all space was filled with them, jammed together every which way. Then it just setted down to two.

"Oooppps,," said Goldwasser. "Have to remember that they are just representatives of their equivalence classes. I don't want to fill up all space with my fumblings. Besides, that would twist everything one dimension, and then we'd be back where we started."

The lab was in darkness, so in the dimness Goldwasser's big form seemed to crouch menacing over the controls. He wore his engineer's boots and his hair long, and a beard as well. He might have been some medieval monk, or primitive witch-doctor, mumbling spells and setting incantations. He touched a knob and set a widget, and checked in his copy of *Birkhoff and MacLane.*

"Now," he said, and played with his instruments. Two new vectors rose out of the intersections. "Cross-products. Now I've a right and a left-handed system."

All the while Gene Gerard was mumbing in the background:

"The Count and Igor his servant took the beautiful maiden down to the crypt. 'Ah, now, my pretty,' snarled the Count. 'Come to my bedchamber, or I'll leave you to Igor's mercies.' The misshapen dwarf cackled and rubbed his paws. 'Decide, decide!' cried the count. His voice was a scream. 'Decide, my dear. SEX — ELSE, IGOR!"

"Augh," said Goldwasser, and shut it off. "Now," he said, "I've got some plasma in the next compartment. I'm going to release it, and shift the ions one dimension. His hands poised over the G. S. generator.

"Holy Halmos," I whispered.

Ted Anderson stood beside the generator. He looked thinner and paler than ever. The metamathematician's acne stood out plainly and I remembered Pearl's remark that he didn't know if he should call Ted a Whitehead or a Blackhead.

Ted smiled, and went into topological convulsions. I looked away, and presently he came back in to shape. "Hard getting used to real space again," he whispered.

"I haven't got long," he said, as that was how he'd planned to start, "so here it is. You know I was working on Ephraim's theories, looking for a flaw. You know the popular explanations of mathenautics. Brill's hypnotherapy and Ephraim's math allow the consciousness to become aware of the abstract structure it put on the universe."

I winced, and he smiled painfully at me. "That's for the kiddies," he said, and I saw that Pearl was standing in the iris of the tube. Johnny was stronger than I'd thought!

"There isn't any flaw," he said. As he did he seemed to grow thin. "Ted, you're rotating," I cautioned.

He steadied, and continued.

"There's no flaw. But the theory is wrong. It's backwards. *This is the real universe,*" he said, and gestured. Beyond the lab topological space remained as always, a blank, the color of the back of your head through your own eyes.

"Now listen to me, Goldy and Johnny and Kidder. What is the nature of intelligence? I guess it's the power to abstract, to conceptualize. I don't know what to say beyond that — and I doubt if I ever will. I don't know what it is. But I know where it came from! Here! In the math spaces — they're alive with thought, flashing with mind!

"When the twiddles circuits failed I cracked. I fell apart, lost faith in it all. For I had just found what I though was a basic error in theory. I died, I vanished . . .

"But I didn't. I'm a metamathematician. An operational philosopher, you might say. I may have gone

mad — but I think I passed a threshold of knowledge. I understand . . .

"People think that math and science people are cranks — fellows that can only get passionate about a differential equation, like that Seidmann fellow. But it isn't like that at all. Math and science are beautiful and mysterious arts. The theory of infinity —

"They're out there. The things we thought we'd invented ourselves. The concepts and the notions and the pure structures — if you could see them . . ."

He looked around the room, desperately. Pearl was rigid against the iris of the tube. Goldy looked at Ted for a moment, then his head darted from side to side. His hands whitened on the controls . . .

"Jimmy," Ted said.

I didn't know. I moved towards him, across the lab to the edge of topological space, and beyond the psychic ecology . . .

"No time, no space, no matter. "But how can I say it? How many people can stay awake over a book of modern algebra, and how many of those can understand?

— I saw a set bubbling and whirling, then take purpose and structure to itself and become a group, generate a second unity element, mount itself and become a group, generate a second unity element, mount itself and become a field, ringed by rings. Near it, a mature field, shot through with ideals, threw off a splitting field in a passion of growth, and became complex.

—I saw the life of the matrices; the young ones sporting, adding and multiplying by a constant, the mature ones mating by composition: male and female make male, female and male make female — sex through anticommutivity! I saw them grow old, meeting false identities and loosing rows and columns into nullity.

— I saw a race of vectors, losing their universe to a newer race of tensors that conqured and humbled them.

—I watched the tyranny of the Well Ordering Principle, as a free set was lashed and whipped into structure. I saw a partially ordered set, free and happy, broken before the Axioms of Zemelo.

— I saw the point sets, with their clicks and clubs, infinite numbers of sycophants clustering round a Bolzano — Weirstrauss aristocrat — the great compact medieval coverings of infinity with denumerable shires — the conflicts as closed sets created open ones, and the other way round.

—I saw the rigid castes of a society of transformations, orthogonal royalty, inner product gentry, degenerates — where intercomposition set the caste of the lower on the product.

— I saw the proud old cyclic groups, father and son and grandson, generating the generations, rebel and blacksheep and hero, following each other endlessly. Close by were the permutation groups, frolicking in a way that seemed like the way you sometimes repeat a sentence endlessly, stressing a different word each time.

There was much I saw that I did not understand, for mathematics is a deep, and even a mathenaut must choose his wedge of specialty. But that world of abstractions flamed with a beauty and meaning that chilled the works and worlds of men, so I wept in futility.

Presently we found ourselves back in the lab. I sat beside Ted Anderson and leaned on him, and I did not speak for fear my voice would break . . .

Anderson talked to Johnny and Ed.

"There was a — a race, here, that grew prideful. It knew the Riemann space, and the vector space, the algebras and the topologies, and yet it was unfulfilled. In some way — oddly like this craft," he murmured, gesturing — "they wove the worlds together, creating the real universe you knew in your youth.

"Yet still it was unsatisfied. Somehow the race yearned so for newness that it surpassed itself, conceiving matter and energy and entropy and creating them.

"And there were laws and properties for these: inertia, speed, potential, quantumization. Perhaps life was an accident. It was not noticed for a long time, and proceeded apace. For the proud race had come to know itself, and saw that the new concepts were . . . flawed." Anderson smiled faintly, and turned to Ed.

"Goldy, remember when we had Berkowitz for algebra," he asked. "Remember what he said the first day?"

Goldwasser smiled. "Any math majors?

"Hmm, that's good.

"Any physics majors?

"Physics majors! You guys are just super engineers!

"Any chemistry majors?

"Chemistry major! You'd be better off as a cook!"

Ted finished, "And so on, down to the, ahem, baloney majors."

"He was number happy," said Ed, smiling.

"No. He was right, in a way," Ted continued. "The race had found its new notions were crudities, simple copies of algebras and geometries past. What it thought was vigor was really sloth and decay.

"It knew how to add and multiply, but it had forgotten what a field was, and what commutivity was. If entropy and time wreaked harm on matter, they did worse by this race. It wasn't interested in expeditions though the fiber bundles; rather it wanted to count apples.

"There was conflict and argument, but it was too late to turn back. The race had already degenerated too far to turn back. Then life was discovered.

"The majority of the race took matter for a bride. It's esthetic and creative powers ruined, it wallowed in passion and pain. Only remnants of reason remained.

"For the rest, return to abstraction was impossible. Time, entropy, had robbed them of their knowledge, their heritage. Yet they still hoped and expended themselves to leave, well, call it a 'seed' of sorts."

"Mathematics?" cried Pearl.

"It explains some things," mused Goldwasser softly. "Why abstract mathematics, developed in the mind, turns out fifty years or a century later to accurately describe the physical universe. Tensor calculus and relativity, for example. If you look at it this way, the math was there first."

"Yes, yes, yes. Mathematicians talked about their subject as an art form. One system is more 'elegant' than another if its logical structure is more austere. But Occam's Razor, the law of simplest hypothesis, isn't logical.

"Many of the great mathematicians did their greatest work as children and youths before they were dissipated by the sensual world. In a trivial sense, scientist and mathematicians most of all are described as 'unworldly'..."

Anderson bobbled his head in the old familiar way. "You have almost returned," he said quietly. "This ship is really a heuristic device, an aid to perception. You are on the threshhold. You have come all the way back."

The metamathematician took his notebook, and seemed to set all his will upon it. "See Ephriam gets this," he murmured. "He, you, I... the oneness —"

Abruptly he disappeared. The notebook fell to the floor.

I took it up. Neither Ed nor Johnny Pearl met my eyes. We may have sat and stood there for several hours, numbed, silent. Presently the two began setting up the isomorphomechanism for realization. I joined them.

IV

The National Mathenautics and Hyperspace Administration had jurisdiction over civilian flights then, even as it does today. Ted was pretty important it seemed. Our preliminary debriefing won us a maximum security session with their research chief.

Perhaps, as I'd thought passionately for an instant, I'd have done better to smash the immy, rupture the psychic ecology, let the eggshell be shattered at last. But that's not the way of it. For all of our progress, some rules of scientific investigation don't change. Our first duty was to report back. Better heads than ours would decide what to do next.

They did. Ephraim Cohen didn't say anything after he heard us out and looked at Ted's notebook. Old Ice Cream sat there, a big teddy-bear-shaped genius with thick black hair and a dumb smile, and grinned at us. It was in Institute code.

The B.C.N.Y. kids hadn't seen anything, of course. So nobody talked.

Johnny Pearl married a girl named Judy Shatz and they had fifteen kids. I guess that showed Johnny's views on the matter of matter.

Ed Goldwasser got religion. Zen-Judaism is pretty orthodox these days, yet somehow he found it suited him. But he didn't forget what had happened back out in space. His book, *The Cosmic Mind,* came out last month, and it's a good summation of Ted's ideas, with a minimum

of spiritual overtones.

Myself. Well, a mathematician, especially a topologist, is useless after thirty, the way progress is going along these days. But *Dim-Dustries* is a commercial enterprise, and I guess I'm good for twenty years more as a business man.

Goldwasser's Grahm - Schmidt generator worked, but that was just the beginning. Dimensional extension's made Earth a paradise, with housing hidden in the probabilities and automated industries tucked away in the dimensions.

The biggest boon was something no one anticipated. A space of infinite dimensions solves all the basic problems of modern computer circuit design. Now all components can be linked with short electron paths, no matter how big and complex the device.

There have been any number of other benefits. The space hospitals, for example, where topological surgery can cure the most terrible wounds — and topological psychiatry the most baffling syndromes. (Four years of math is required for pre-meds these days.) Pata-psychology and pata-sociology finally made some progress, so that political and economic woes have declined — thanks too to the spaces, which have drained off a good deal of poor Earth's over-population. There are even spaces resorts, or so I'm told — I don't get away much.

I've struck it lucky. Fantastically so.

The Private Enterprise Acts had just been passed, you'll recall, and I had decided I didn't want to go spacing again. With the training required for the subject, I guess I was the only qualified man who had a peddler's pack too. Jaffee, one of my friends down at Securities and Exchange, went so far as to say that *Dim-Dustries* was a hyperspherical trust (math is required for pre-laws too.) But I placated him and I got some of my mathemateers to realign the Street on a moebius strip, so he had to side with me.

Me, I'll stick to the Earth. The "real" planet is a garden spot now, and the girls are very lovely.

Ted Anderson was recorded lost in topological space. He wasn't the first, and he was far from the last. Twiddles circuits have burned out, Naught-Am-Revs have gone mad, and no doubt there have been some believers who have sought out the Great Race.

Ted, I did what I thought was best... END